

# Statistical Analysis of Under-sampled Dynamic Displacement Measurement

Daniele Inaudi <sup>1,2</sup>, Joel Pascal Conte <sup>3</sup>, Nicholas Perregaux<sup>1</sup>, Samuel Vurpillot <sup>1</sup>

## <sup>1</sup> IMAC - Stress Analysis Laboratory

Swiss Federal Institute of Technology, Department of Civil Engineering  
CH-1015 Lausanne, Switzerland

## <sup>2</sup> SMARTEC SA

via al Molino 6  
CH-6916 Grancia, Switzerland

## <sup>3</sup> University of California Los Angeles

School of Engineering and Applied Science

### 1. ABSTRACT

The Lutrive highway bridges are two twin bridges built in 1972 by the cantilever method with mid-span articulations. The bridge deck consists of a box girder of variable depth. In 1997 this bridge was instrumented with 30 SOFO fiber optic deformation sensors installed inside the box girder. This sensor network is mainly aimed at measuring the short- and long-term spatial displacements of one of the spans. This includes the displacement resulting from the daily variations of temperature and the long-term creep effects. It was found that these same sensors could also be used to capture the quasi-static part of the dynamic deformation of the bridge under traffic load. Although the measurement system can acquire measurements only at intervals a few seconds apart, it was found that these "snapshots" could give interesting information about the low frequency quasi-static deformations of the bridge. The data from pair of sensors was combined to obtain information about the instantaneous curvature variations of the bridge. This curvature data was then analyzed statistically to extract information about the dynamic traffic loads. This measurement and analysis method was validated in a fatigue test on a concrete slab.

**Keywords:** Deformation sensor, Dynamic testing, Structural monitoring, Fiber optic sensors.

### 2. INTRODUCTION: STATIC AND DYNAMIC MEASUREMENTS

The importance of monitoring all structures of some significance is an evidence [1,2]. Monitoring is fundamental in order to guarantee the safety of a structure and its users (think of a dam, a bridge or a tunnel). It also helps in the planning of maintenance intervention and to increase the knowledge of its real behavior, permitting the optimization of future similar structures.

When deformation monitoring is considered, two main long-term monitoring approaches are possible: static and dynamic. In the static case, the long-term deformations of the bridge are measured with a stable system capable of comparing the deformations at different times spaced by long periods (typically years). On the other side, dynamic analysis measures bridge degradations comparing its dynamic behavior at different times. In this case, measurements obtained over relatively short periods are used to identify changes in the static system of the bridge (system identification).

Our group has developed and successfully applied a static deformation monitoring system called SOFO. It is based on low-coherence interferometry [3] and is composed by a portable reading unit [4,5,6] adapted to building yard conditions, different types of sensors adapted to installation in or on most construction materials [7,8] as well as a number of software packages

allowing the storage and the computer aided analysis of a large number of measurements [9]. All these components are now commercially available through the company SMARTEC SA.

Although this system proved very reliable for the long-term static monitoring of structures like bridges, tunnels, dams, geostructures and historical buildings, it is not directly adapted to dynamic measurements since a single measurement requires about 10s to be carried out. A new research project was started at IMAC to develop a new reading unit capable to demodulate the standard SOFO sensor at higher rates (typically 100 Hz to 10 kHz). As shown in this paper it is however possible to obtain interesting information about the dynamical behavior of a structure by statistically analyzing the data obtained with the current static SOFO system.

### 3. STATISTICAL ANALYSIS OF UNDERSAMPLED DYNAMIC DATA

The current SOFO system is capable of measuring deformations with a precision of 2 microns over measurement bases between 20 cm and 10 m. The reading unit delivers a deformation reading each 10 s. If the measurement is performed on a structure undergoing dynamic deformations, the reading unit will record a snapshot of its instantaneous deformations each 10 s. If the deformations of the structure are statistically repeatable, as in the case of periodic loading or traffic loading, it is possible to gain interesting information about the structural dynamics by statistically analyzing a large number of these snapshots.

If the structure undergoes a time -dependent deformation  $\Delta L(t)$  it is possible to associate a probability function  $P(\Delta L)$  describing the relative probability of finding the structure in a state of deformation  $\Delta L$  at any given time.

In the following two experimental examples we will show how it is possible to obtain the  $P(\Delta L)$  function by analysis of the static SOFO measurements

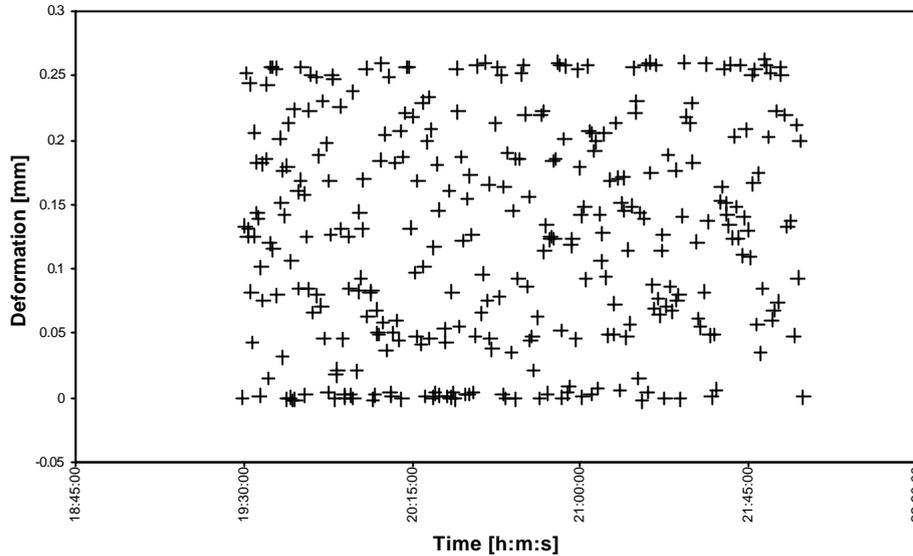
### 4. APPLICATION EXAMPLE: PERIODIC DEFORMATIONS

To test the statistical analysis method, we started with a concrete beam under fatigue testing (see Figure 1). The beam was subject to a sinusoidal loading in four-point-bending setup with a frequency of about 4 Hz. Four SOFO sensors were imbedded in the beam at different heights and were used to characterize the static residual deformations after an increasing number of cycles [10].



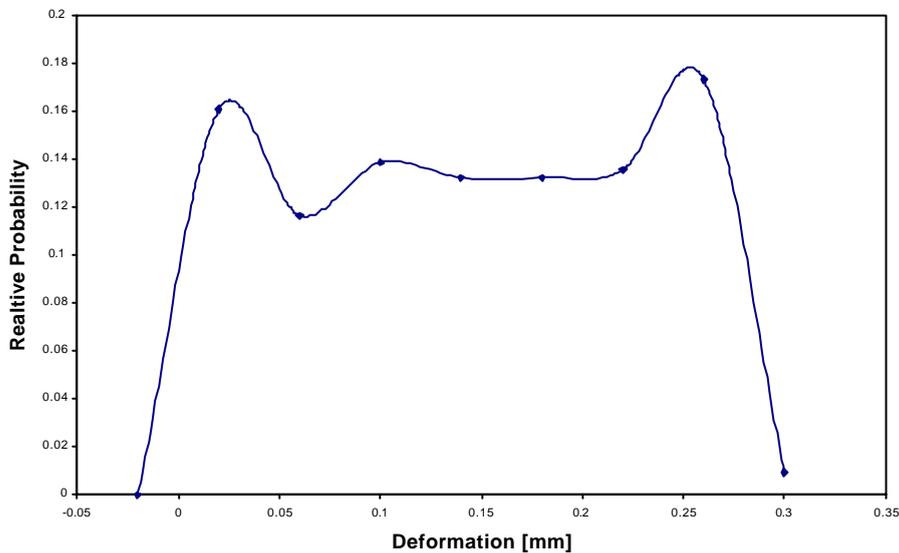
Figure 1: Concrete beam subject to cyclic four-point-bending. The SOFO system is visible in the foreground.

Figure 2 shows the deformations measured on the beam while the sinusoidal deformation was applied. The noise-like data are in fact snapshots of the beam taken each 10 s.



**Figure 2: Deformation data on a dynamically loaded concrete beam.**

This figure clearly shows that the deformation is bounded between 0 and about 0.25 mm. If we now plot a histogram of these 317 measurement points we obtain the desired probability function. This is shown in Figure 3.



**Figure 3: Relative deformation probability for the data in Figure 2.**

As expected for a sinusoidal loading the maximal probability is found for the extreme deformations. This corresponds to the positions where the beam spends most of its cycling time. The amplitude of the deformation is comparable to the 0.26 mm deformation measured under a static load corresponding to the maximum imposed dynamic load. This shows that dynamic

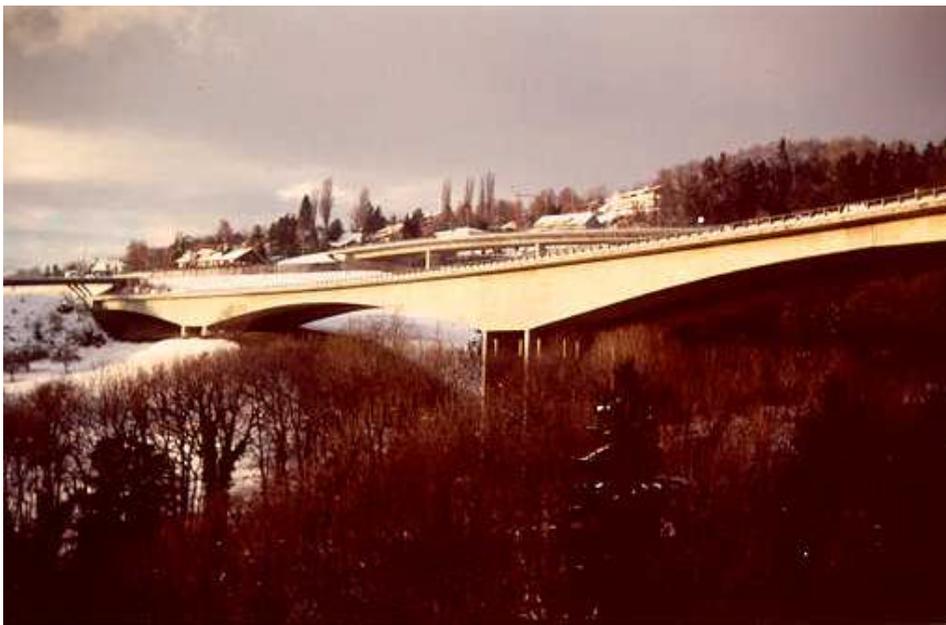
effects do not amplify the beam deformations. This is expected since the excitation frequency lies well below the resonance frequency of the beam.

## 5. APPLICATION EXAMPLE: TRAFFIC LOADING

The Lutrive (Switzerland) North and South bridges are two parallel twin bridges (see Figure 4). Each one supports two lanes of the Swiss national highway RN9 between Lausanne and Vevey. Built in 1972 by the corbelling method with central articulations, the two bridges are gently curved ( $r = 1000$  m) and each bridge is approximately 395 m long on four spans. The two bridges have the same cross-section. It consists of a box girder of variable height (from 2.5m to 8.5m) and two slightly asymmetric cantilevers meant to reduce the effect of torsion in the curved bridges.

The fourth span of the South bridge, fitted with an hydrostatic leveling system measuring vertical displacements since 1988, was instrumented with 30 6m long SOFO sensors. To measure the curvature variations, the sensors are installed in pairs at the interior of the box girder. Curvatures are measured with sensors placed near the top and the bottom of bridge web and the vertical displacements can be retrieved by double integration of the curvatures [11]. The sensors were used mainly for quasi-static testing under thermal loading and under a static load-charging test.

In order to test the statistical analysis algorithm, two sensors placed at quarter-span (see Figure 5) where measured during 24 hours under traffic loading. Both sensors were measured with a single scan of the SOFO system by coherence multiplexing them with an external coupler. The data from the two sensors is therefore correlated since the two readings are obtained in less than 0.1 s.



**Figure 4: The Lutrive bridge.**

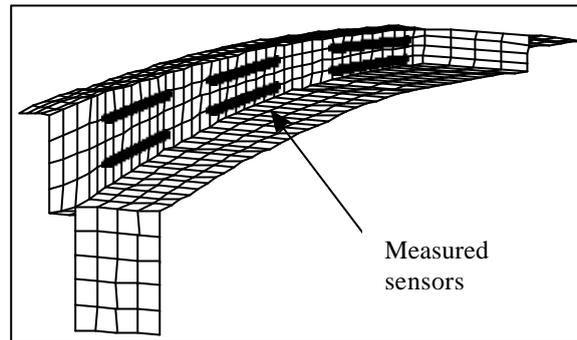


Figure 5: SOFO sensors placement

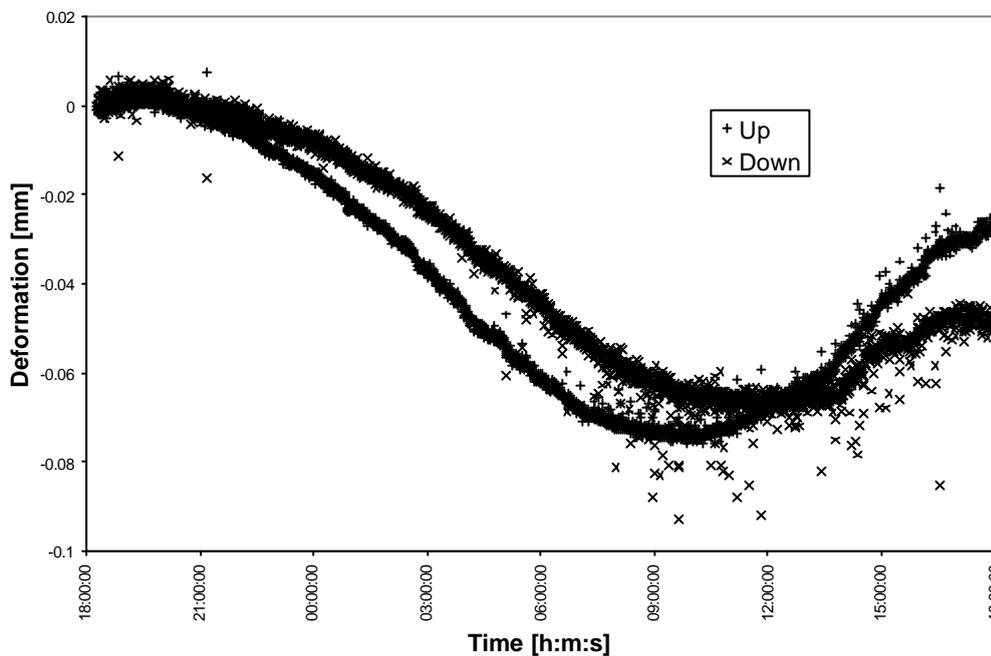
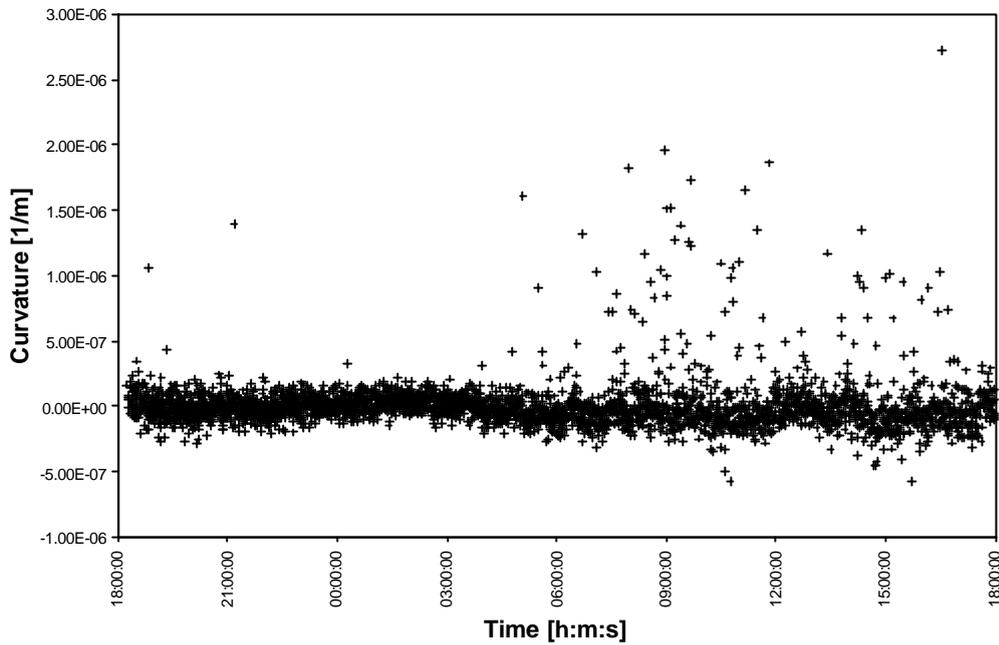


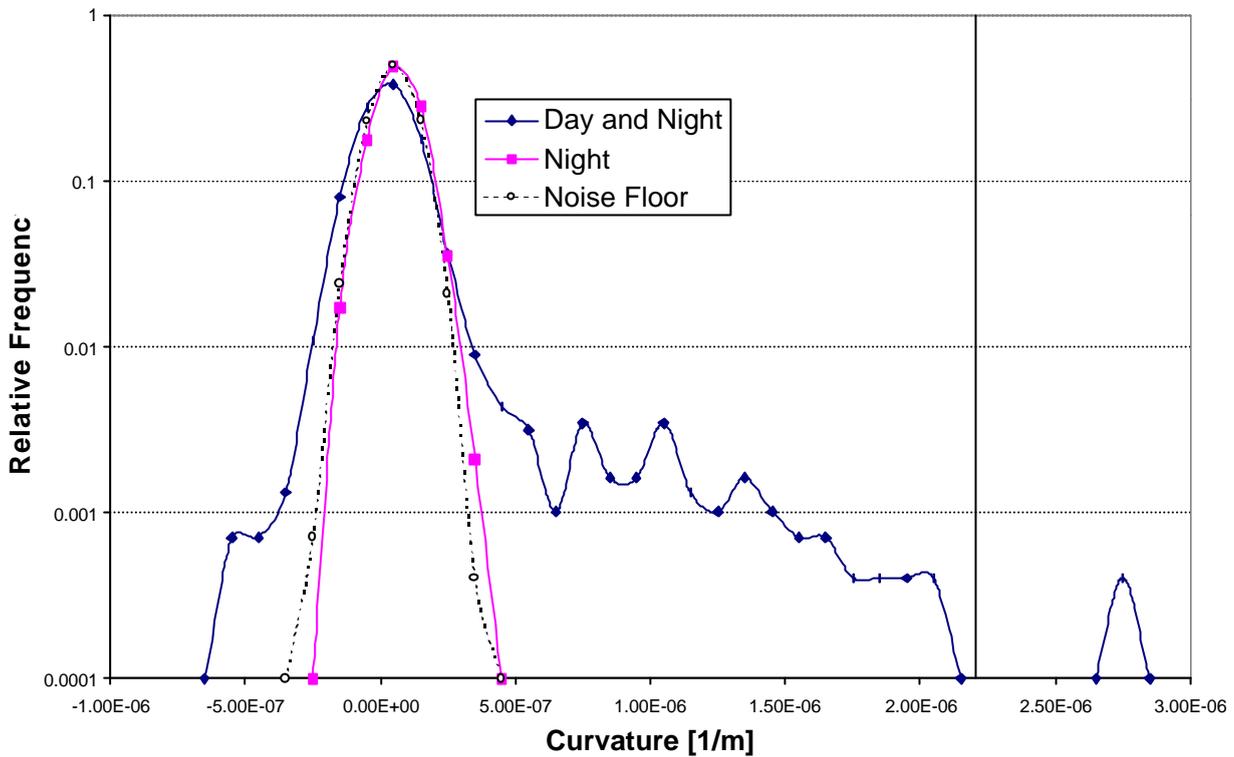
Figure 6: Deformation data obtained on two parallel sensors placed at quarter-span in the Lutrive bridge.

Figure 6 shows the results obtained by the two sensors. The drift in the measurements on both sensors is due to the bridge's temperature variations that were however very small in this cold winter day. The total drift is of only about 80 microns over a sensor length of 6 m. This corresponds to a temperature variation of the order of 1°C. By subtracting the two deformation values and dividing by their length and distance it is possible to obtain the curvature variations of the bridge.



**Figure 7: Curvature data after removing the bridge's temperature drift.**

Figure 7 shows the curvature readings after removal of the bridge's temperature drift with a polynomial fit on the initial data. Has expected most data points lie around a zero curvature corresponding to the mean static state of the bridge. High curvature values indicate a deformation induced by the passage of a truck. It can be noticed that these points are concentrated during the day when truck circulation is allowed. Except for one reading around 21:00 no trucks were registered during the night. Most curvatures are positive and indicate a downward bending of the bridge under the quasi-static loading of the truck. A few points lie outside the noise floor curve for negative curvatures. These ere either rebounds of the bridge after a truck leaves the instrumented span or deformations induced by trucks on neighboring spans.



**Figure 8: Relative curvature frequency of the data in Figure 7.**

Figure 8 show the statistical analysis on the whole set of data (Day and Night, 3269 data points), the night data only (Night, 996 points) and of a Monte-Carlo simulation based on simulated deformation measurements with a standard deviation of one micron (noise Floor, 3267 points). The good agreement between these two latter curves indicates that the noise floor of the SOFO system limited the measurements during the night. During the day the higher deformations due to the passage of the trucks are clearly visible. The vertical line indicates the curvature that was obtained during the static loading test with a truck of 28 t placed at center span. Since the load limit on the Swiss highway network is 28 t, Figure 8 indicates that the dynamic effects on the bridge are very limited. A rebound of a passing truck could for example induce higher curvatures than this same truck statically placed on the bridge. The isolated event at  $2.7 \cdot 10^{-6}$  1/m either indicates an overweight truck (some 40 t trucks are also circulating) or two trucks on the bridge at the same time.

This same figure can be used to quantify the probability of having a truck on the bridge at any give time. This is given by the ratio of the number of points of the Day and Night curve inside, respectively outside the Noise Floor curve. This ratio gives 90% and indicates the probability of obtaining a reliable static deformation value when the measurements are performed under traffic conditions. Therefore it can be considered sufficient to perform three measurements on each sensor and discard any statistically aberrant value to obtain a reliable value. A theoretical analysis considering the truck traffic density, the mean truck speed, the mean transit time gives a slightly lower value of 80%.

## 6. CONCLUSIONS

The presented experiments show the possibility of using a relatively slow data acquisition system to statistically analyze dynamic deformations of structures. The only requirements are that the deformation measuring system is able to take instantaneous snapshots of a moving structure and that a sufficient number of data points are available for analysis. The results also show that it is possible to obtain reliable static data on a bridge under traffic conditions.

## 7. ACKNOWLEDGEMENTS

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