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Vertical Deflection of a Pre-Stressed Concrete Bridge Obtained Using Deformation Sensors and Inclinometer Measurements





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The serviceability of a bridge is generally analyzed by a comparison between the vertical deflections expected by the engineer and those measured during a load test or in the long term. The existing methods do not allow the determination of the vertical displacements from the measurements carried out by a network of deformation sensors placed inside the bridge. The mathematical model presented here allows the determination of the displacement field from internal horizontal deformation measurements and helps in the design of the required sensor network. This model was tested on an experimental model and on the Lutrive Highway Bridge (Switzerland) by comparing the changes in vertical displacements under daily temperature variations obtained with the proposed method, with those measured directly using an absolute hydrostatic leveling system. Fiber optic deformation sensors and electrical inclinometers were used to carry out the measurements. With this deformation monitoring system, featuring a precision of 10 μm on 1 m long deformation sensors, it is possible to retrieve the vertical displacement field of a beam with a global error less than 8 percent.

Keywords: bridge monitoring; deformation; deformation sensors; fiber optic sensor; long-term monitoring; smart structures; structural analysis.

RESEARCH SIGNIFICANCE

In many bridges, the vertical displacements are the most relevant parameters to be monitored in both the short and long term. Current methods (such as triangulation, water levels, or mechanical extensometers, etc.) are often tedious to use and require the intervention of specialized operators. The resulting complexity and costs limit the temporal frequency of these traditional measurements. The spatial resolution obtained is in general low, and only the presence of anomalies in the global structural behavior can be detected and warrant a deeper and more precise evaluation.

To measure bridge vertical displacements at low cost and frequency in time, a solution consists of installing a network of relative displacement sensors during concrete pouring or installing it on the surface of the structure.

This paper reports the results of one of the first full-scale tests on a bridge with fiber optic sensors, and will help engineers to choose the sensor network design.

INTRODUCTION

The security of certain large civil structures demands a periodical monitoring in order to ensure the safety of its users and to help in the planning of the maintenance interventions. The current methods (such as triangulation, hydrostatic leveling, vibrating strings, or mechanical extensometers) are often tedious in their application and require the intervention of specialized operators. The resulting complexity and costs limit the frequency of these measurements in time. The spatial resolution obtained is usually low and only the presence of anomalies in the global structural behavior urges a deeper and more precise evaluation. A quality network of sensors equipping the structure does not increase by itself its safety if not accompanied by a comprehensive analysis of the measurements. There is, therefore, a real need for a tool enabling automatic and permanent structural and satisfying the attributes of high precision and good spatial resolution. In the last few years, the concept of "smart structures" has increasingly gained in interest from the civil engineering community. These structures composed of sensors, processing units, and actuators could provide an answer to these needs.

MATHEMATICAL MODEL FOR THE DETERMINATION OF VERTICAL DISPLACEMENTS FROM INTERNAL HORIZONTAL DEFORMATION MEASUREMENTS IN A BEAM

Most embedded deformation sensors (e.g. SOFO system,[1] micro-bending sensors, inductive sensors, etc.) measure the relative displacement between two points inside the structure. The analysis of these measurements, generally obtained with sensors placed horizontally in a bridge, is not straightforward. In order to determine the overall displacement field

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of the structure from internal deformation measurements, the present algorithm uses the relation between the vertical and the curvature of a linear prismatic beam element (Fig. 1).

Deformation of a beam: review

Considering the plane section conservation law of Bernoulli, the vertical displacements of a uniformly loaded beam on n spans is expressed as a sequence of n fourth degrees polynomial with a C_1 continuity at their border.† Each polynomial $p_i^4(x)$ domain includes a section of beam which has a constant inertia, a constant uniform load, and end forces and moments. The second derivative of the vertical displacement gives n second degree polynomials. To determine the exact displacement functions, it is therefore necessary to retrieve the curvature functions $P_i^2(x)$ on the beam sections and to integrate them twice,‡ guaranteeing the C_1 continuity on the boundaries.

Curvature measurement

According to Bernoulli's assumption, the elongational strain at any material fiber and the beam curvature are related as

$$\frac{1}{r(x)} = -\frac{\varepsilon(x)}{y} \tag{1}$$

where

r = radius of curvature
x = curvilinear abscissa along the beam
ε = elongational strain
y = distance from the neutral axis

A relative displacement gauge, installed parallel to the neutral axis, measures the elongational deformation of a fiber of length l_1 (see Fig. 2). The integration of Eq. (1) gives

$$\int_0^{l_1}\frac{dx}{r(x)} = -\int_0^{l_1}\frac{\varepsilon(x)dx}{y} \Rightarrow \frac{1}{l_1}\int_0^{l_1}\frac{dx}{r(x)} = -\frac{1}{l_1}\frac{\int_0^{l_1}\varepsilon(x)dx}{y} \Rightarrow \frac{1}{r_m} = \frac{-(l_2-l_1)}{y.l_1} \tag{2}$$

where

r_m = mean radius of curvature
l_1 = initial sensor fiber length
l_2 = final sensor fiber length

Equation (2) shows that a relative displacement gauge, placed parallel to the neutral axis measures the mean curvature $\frac{1}{r_m}$ of the element of the beam. In the case of combined bending and axial load and temperature variations, it can be shown that a pair of relative displacement gauges, placed at different distances parallel to the neutral axis are required to measure the mean curvature of a beam element.

Curvature function of the beam sections*

The curvature function of each beam section is a second degree polynomial of the form

$$P^2(x) = ax^2 + bx + c \tag{3}$$

Since the polynomial $P^2(x)$ has three unknowns, only three independent measurements are necessary to determine these three coefficients for a single beam section.

With three relative displacement sensors, we obtain the following information (see Fig. 3):

$$\frac{1}{r_1}: \text{mean curvature on } [x_1'; x_1'']$$

$$\frac{1}{r_2}: \text{mean curvature on } [x_2'; x_2'']$$

$$\frac{1}{r_3}: \text{mean curvature on } [x_3'; x_3'']$$

† C_1 border continuity: The function and its derivative are continuous at the borders.

‡ The curvature is equal to $\frac{1}{r} = \frac{\frac{d^2v}{dx^2}}{\left[1+\left(\frac{dv}{dx}\right)^2\right]^{\frac{3}{2}}}$ for the displacements of in civil structures, $1+\left(\frac{dv}{dx}\right)^2 \cong 1$; therefore, for the rest of the paper, we consider that $\frac{1}{r} = \frac{d^2v}{dx^2}$.

* Beam section: A segment of the whole beam with a constant inertia, a constant uniform load and an additional introduction of concentrated load (force, moment, support reaction,...) only at its extremes. For example, a beam with two concentrated loads can be divided into 3 sections; a beam uniformly loaded on 3 supports can be divided into 2 sections.

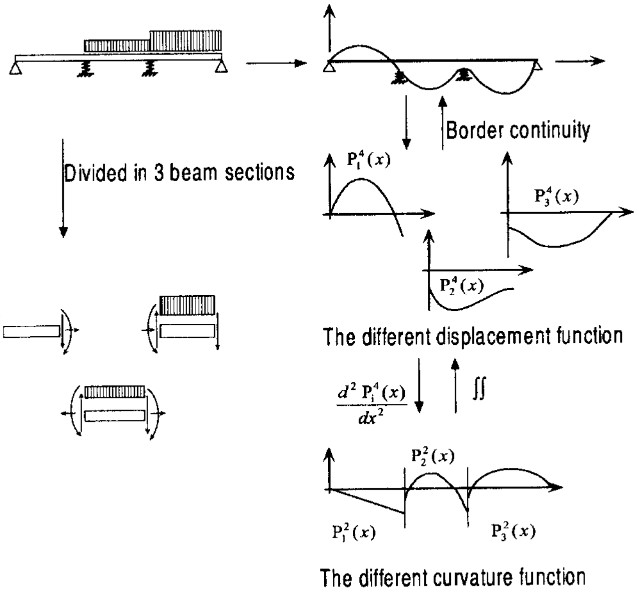

Fig. 1—Relation between load, displacement, and curvature

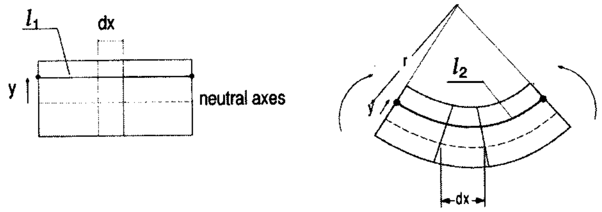

Fig. 2—Deformation of a relative displacement gauge for pure bending

where x_i' and x_i'' denote the left limit and the right limit, respectively, of sensor i.

The coefficients a, b, c are solutions of the following linear system of equations:

$$\frac{\int_{x_i'}^{x_i''}(ax^2 + bx + c)dx}{x_i'' - x_i'} = \frac{1}{r_i},\ i = 1, 2, 3$$

Displacement function of beam

Equation (3) gives the curvature function of the adjacent beam sections. We retrieve the displacement functions by integrating them twice. Furthermore, the C_1 continuity of the displacement functions must be guaranteed at the borders. The displacement functions are expressed as

$$P_i^4(x) = \iint P_i^2(x)dx' + \alpha_i x + \beta_i,\ i = 1, 2, ..., n \quad (4)$$

where the constants of integration α_i, and β_i, are obtained by enforcing the following continuity conditions:

$P_i^4(X = L_i) = P_{i+1}^4(X = 0)\big|_{i \in [1;n-1]}$ (expression of displacement continuity between two adjacent beam sections),

n = number of beam sections;

$P_i^{4\prime}(X = L_i) = P_{i+1}^{4\prime}(X = 0)\big|_{i \in [1;n-1]}$ (expression of slope continuity between two adjacent beam sections);

$P_1^4(X = 0) = 0 \qquad P_n^4(X = L_n) = 0$ (expression of zero displacement boundary condition at both ends of the beam). (5)

There are two unknowns for the displacement field of each of the n beam sections (see Eq. [4]). The above continuity and boundary conditions give $2n$ equations from which the $([n\text{-}1] + [n\text{-}1]) + 2 = 2n$ unknowns can be obtained uniquely.

Discussion

We have determined that only three relative displacement sensors, positioned at different locations in each beam section, are sufficient to determine the displacement field of the whole beam. The beam is subdivided in sections of constant inertia and elastic response, uniformly loaded and with additional concentrated loads (force, moment, support reaction, etc.) only at their extremities.

The above general hypotheses follows Bernoulli-Navier beam theory and require the knowledge of the boundary conditions of the whole beam (see Eq. [5]). The Bernoulli hypothesis is usually satisfied under serviceability condition, while the boundary conditions are known in general. Engineers are interested in the beam's internal forces. These forces are induced only by the relative displacements of the beam with respect to its chord. This means that the above methodology is able to extract the deformation of the beam but not its rigid-body displacements in space (see Fig. 4). To obtain information about these displacements, internal sensors are obviously useless and other measurements relative to fixed external points obtained using absolute sensors (such as GPS, inclinometer, etc.) should be carried out. The minimum number of sensors to be placed in the structure depends on the number of parameters needed to retrieve its curvature. One sensor is sufficient for the case of pure bending (constant bending moment diagram), a couple of sensors are necessary for the case of bending combined with axial force, and one additional sensor per section of beam is sufficient in the case of a linearly varying distributed load.

In a real prestressed concrete bridge with varying moment of inertia, nonuniform load, etc., it is necessary to determine the form of the curvature diagram (for example, through finite element analysis). Generally, the curvature diagram can be approached by a second degree polynomial function. Assuming that the position of the neutral axis is unknown, three pairs of sensors could then be sufficient to determine the vertical displacement of the bridge (see example of the Lutrive Highway Bridge). An increased number of sensors will enhance the measurement precision through a least square fitting algorithm and add some redundancy useful in the case of sensor failures.

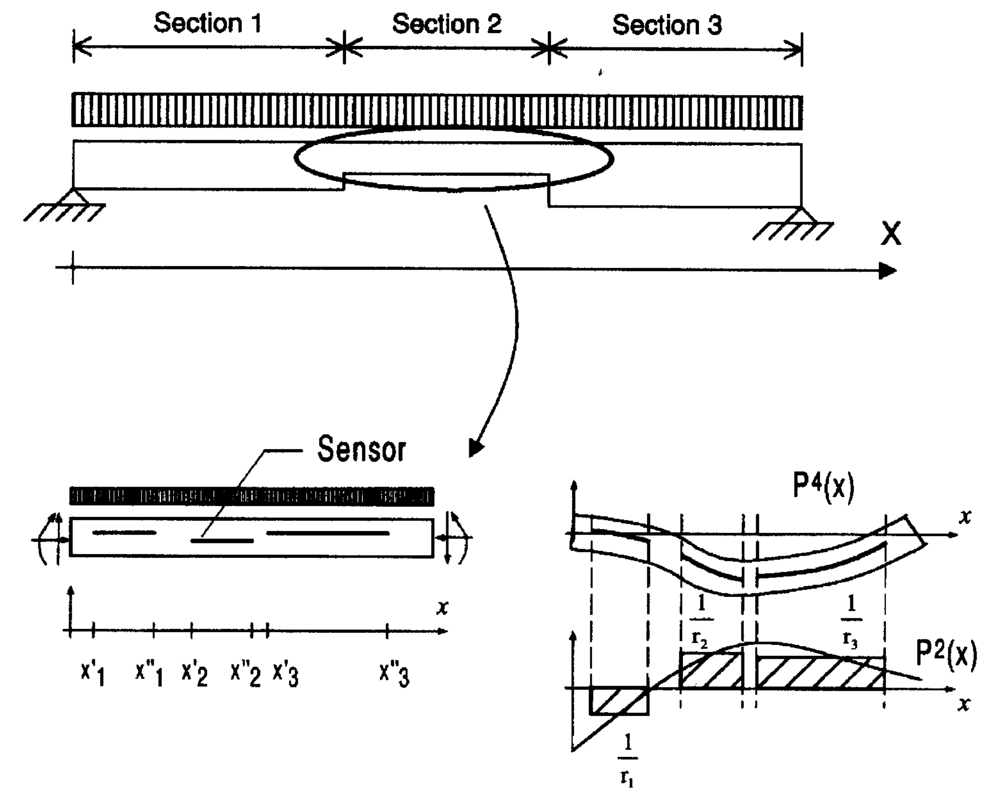

Fig. 3—Curvature function measured for a beam section (e.g. Section 2)

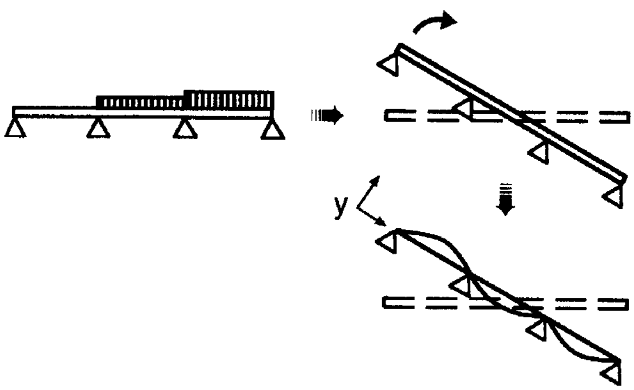

Fig. 4—Separation between rigid body and relative displacements

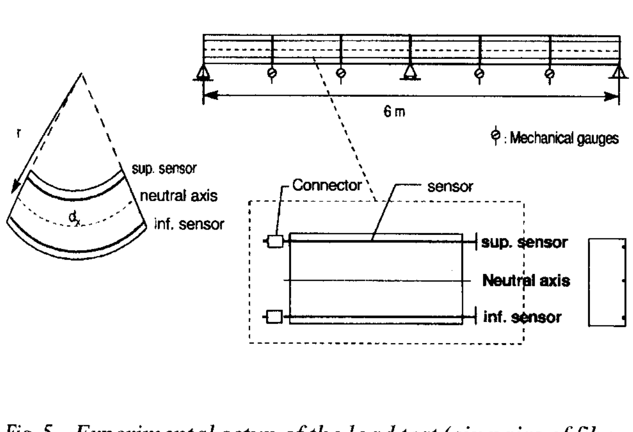

Fig. 5—Experimental setup of the load test (six pairs of fiber optic sensors equip the whole beam)

TEST OF THE ALGORITHM ON A TIMBER BEAM

A test has been carried out on a 6 m long timber beam with a 112×156 mm^2 section on three supports. It was equipped with 12 fiber optic deformation sensors of the SOFO system.[1]

The main beam is installed on three supports, and was thus subdivided in two beam sections. To retrieve the beam overall deformation, the average curvature had to be measured in each beam section of beam at three different locations. Each section was therefore equipped with three pairs of fiber optic sensors. Two optical fiber sensors of a given pair measured the elongation and shortening above and below the neutral axis. Assuming a constant temperature, only one sensor per cell would have been necessary (see Fig. 5).

The beam deformation calculated by the algorithm presented above was compared to the beam deformation monitored by four mechanical gauges installed under the beam. The central support, not explicitly introduced in the algorithm, provided another point of comparison. The beam is loaded uniformly on one span and 7 mm of vertical deflection corresponds to the prescription imposed for most civil beam type structures.

Figure 6 shows the comparison between the displacement obtained through the present algorithm using the fiber optic sensor outputs and those measured directly by mechanical gauges. The results indicate a good agreement (global error less than 8 percent). The existence of the central support is also retrieved by the proposed algorithm.

TEST OF THE ALGORITHM ON A HIGHWAY BRIDGE

To test the feasibility of the curvature measurements on a full-scale structure, a highway bridge was selected. Since it